

CSE 6311
Homework 2

Problem 1: Consider the following “nearest neighbor” heuristic for the metric travelling salesman problem: Start with any vertex, say v_1 . Then go to the nearest vertex from v_1 , say v_2 . Then go to the nearest remaining vertex to v_2 , say v_3 . Continue until all vertices have been visited, and finally return to v_1 .

Analyze the approximation factor of this algorithm.

Problem 2: Consider the following “closest point” heuristic for the metric TSP problem. Begin with a trivial cycle consisting of a single arbitrary vertex. At each step, identify a vertex u not on the cycle whose distance to any vertex on the cycle is minimum. Let the nearest vertex to u on the cycle be v . Extend the cycle by inserting u just after v .

Analyze the approximation factor of this algorithm.

Problem 3: In the greedy heuristic for the set covering problem, we encountered a summation of the form $H(n) = 1/1 + 1/2 + 1/3 + \dots + 1/n$.

Give a formal proof that $H(n) = O(\log n)$.

Problem 4: Consider a restricted version of the set cover problem where each set has only at most a constant number of elements, say c . What is the approximation factor of the greedy set cover heuristic on this problem? Can this idea be used to produce better approximation algorithms for the vertex cover problem where graphs have degree bounded by a constant?

Problem 5: Give an integer programming formulation of the 3SAT problem.

Problem 6: Look up the definition of the *knapsack problem*, and design a polynomial time approximation scheme for it.

Problem 6: Someone claims that he has designed an approximation algorithm for the maximum independent set problem that have the following property: there exists a constant C such that his algorithm can produce a solution such that $|\text{approx} - \text{opt}| \leq C$. Prove that unless $P=NP$, his claim is false.